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3D NUMERICAL MODEL OF SECONDARY STREAMING IN AN ACOUSTIC-RESONANCE TUBE REFRIGERATOR.

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Abstract

Acoustic streaming accompanying oscillations of a gas in an acoustic-resonance tube refrigerator is considered. The effect of viscous losses within a stack on the form of acoustic streaming in a viscous heat conducting medium is investigated. A numerical model for streaming is suggested that includes terms derived from an analytical formulation of the acoustic fields. Accurate analytical interface conditions in the cross section separating stack and tube resonator are included.

INTRODUCTION

Linear theory developed during the last decades has explained the working principles of thermo-acoustic engines rather well [1, 2]. This gave a great impulse to the applied investigations and development of thermo-acoustic refrigerators. However, though explaining the basic phenomena, linear theory places aside some essential processes that stem from nonlinearity of the sound field occuring within resonators. As early as in 1975 Merkli and Thomann estimated analytically the heat flux along a closed tube [3] and showed that the non-uniform heating and cooling of the tube walls on near-resonant oscillation is caused by a second order terms in the equations. It implies that understanding the fundamentals of heat and mass exchange in resonators is impossible without investigating second order effects such as secondary streaming. The study of secondary streaming was a new ground, broken by Rayleigh [4], and literature on it became very extensive since that time [5].

Recently, analytical models were derived for acoustic streaming inside open and closed resonators (they can easily be applied to the main resonator as well as to the space between stack plates) [5, 6, 7]. In [6], a model of acoustic streaming in an open tube with a real gas is developed taking account the effects of viscosity and heat conductivity within the boundary layer, whereas in [5], models for acoustic streaming both in open and closed cylindrical tubes and plane channels are developed for an ideal gas in assumption of temperature dependent viscosity. The simplest thermo-acoustic engine is a closed cylindrical tube with a set of plane parallel plates placed in it. The visualization of flow in a device of this kind [8] demonstrated that the stack introduces an essential perturbation in the system and that current analytical models for acoustic streaming fail in the stack area. Therefore, interface conditions should be given in the flow in a cross-section separating a stack and a tube-resonator at both ends of a stack.

In the present work, an attempt is made to simulate numerically acoustic streaming in a thermo-acoustic tube-refrigerator using the results of analytical theory of oscillation in a tube to derive source terms for the flow and analytical continuity conditions at the edges of a stack.

DERIVATION OF EQUATION FOR SECONDARY STREAMING IN A CYLINDRICAL TUBE

Consider oscillation in a long cylindrical tube ($L/R >> 1$, where R is the radius of the tube and L is its length) along the x-axis. The usual technique to obtain secondary streaming is either a perturbation method according to which all thermodynamic quantities can be presented in the form of expansion in series by powers of a small parameter [6] or a method of successive approximation. In the latter, the difference between the total values of thermodynamic quantities and their steady-state values is given by [5]

$$p - p_0 = p_1 + p_2, \quad \rho - \rho_0 = \rho_1 + \rho_2, \quad \mathbf{u} = \mathbf{u}_1 + \mathbf{u}_2, \tag{1}$$

where p_1, ρ_1 and $\mathbf{u}_1$ are the acoustic pressure, density and velocity, p_0 and ρ_0 are the zeroth-order steady-state mean values, p_2, ρ_2 and $\mathbf{u}_2$ are the second-order quantities. This assumption permits to separate Navier-Stokes, continuity and energy equations into sets of equations of the first and the second order. For the first order, based on the assumption that tube is long (or narrow), one can neglect some of the terms and obtain [6]:

$$\rho_0 \frac{\partial u_1}{\partial t} + \frac{\partial p_1}{\partial x} = \frac{\mu}{r} \frac{\partial}{\partial r}\left(r \frac{\partial u_1}{\partial r} \right), \tag{2.a}$$

$$\frac{\partial \rho_1}{\partial t} + \frac{\rho_0}{r} \frac{\partial}{\partial r}(r v_1) + \rho_0 \frac{\partial u_1}{\partial x} = 0, \tag{2.b}$$

$$p_1 = \rho_0 R_g T_1 + R_g T_0 \rho_1, \tag{2.c}$$

$$\rho_0 c_p \frac{\partial T_1}{\partial t} = \frac{\partial p_1}{\partial t} + \frac{\lambda}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_1}{\partial r}\right), \tag{2.d}$$

where T_0 is the mean temperature; T_1 is the amplitude of temperature oscillation; R_g is the universal gas constant; c_p is the specific heat at constant pressure; μ is the dynamic viscosity coefficient; x and r are the axial and radial coordinates; u and v are the axial and radial components of velocity, respectively; t is time. The general solution of (2), in high-frequency approximation (when $H >> 1$, where $H = R\sqrt{\omega/\nu}$ is the frequency parameter, ω is the cyclic frequency of oscillation, ν is the kinematic viscosity coefficient), has the form [6]:

$$p_1 = \rho_0 c_0^2 r_1 \cos(kx + \alpha + i\beta)\exp(i(\omega t + \psi)), \tag{3.a}$$

$$u_1 = r_1 c_0 k \sin(kx + \alpha + i\beta)\exp(i(\omega t + \psi - \pi/2))\left[1 - e^{-(1+i)\eta}\right], \tag{3.b}$$

$$v_1 = \frac{(1+i)\omega\delta}{2} r_1 \cos(kx + \alpha + i\beta)\left\{\left(1 + \frac{\kappa - 1}{\sqrt{\sigma}}\right)\frac{r}{R} - e^{-(1+i)\eta} - \frac{\kappa - 1}{\sqrt{\sigma}} e^{-(1+i)\eta\sqrt{\sigma}}\right\}\exp[i(\omega t + \psi)], \tag{3.c}$$

$$\rho_1 = \rho_0 r_1 \cos(kx + \alpha + i\beta)\left\{1 + (\kappa - 1)e^{-(1+i)\eta\sqrt{\sigma}}\right\}\exp[i(\omega t + \psi)]. \tag{3.d}$$

Here r_1 and ψ are the magnitude and the phase of the dimensionless oscillation amplitude; $k = k_0[(1+\beta') + i\beta'']$ is the complex wave number, $k_0 = \dfrac{\omega}{c_0}$ is the wave number in an ideal gas, c_0 is the sound velocity in an undisturbed gas, $\beta'' = -\beta' = -\dfrac{1}{2}\left(1 + \dfrac{\kappa - 1}{\sqrt{\sigma}}\right)\dfrac{\delta}{R}$, β' is the dispersion coefficient, β'' is the absorption coefficient, $\eta = \dfrac{R - r}{\delta}$ is the dimensionless radial coordinate, $\delta = \sqrt{\dfrac{2\nu}{\omega}}$ is the boundary layer thickness, $\kappa = c_p / c_v$ is the specific heat ratio at constant pressure and volume; α, β, r_1 and ψ are determined from boundary conditions. The complex wave number can be represented in a trigonometric form as $k = k_0 \tilde{r} e^{i\varphi}$ where $\tilde{r} = \sqrt{(1+\beta')^2 + \beta'^2}$, $\varphi = \arctan\left(-\beta' / \sqrt{(1+\beta')^2 + \beta'^2}\right)$.

The equations of the second approximation describe the oscillatory motion with the frequency 2ω and the secondary stationary motion. We shall solve them numerically therefore we do not need to reduce them by neglecting of higher order terms and they are given here in the form they are used in simulations. To separate the stationary motion, we average these equations over time:

$$\frac{1}{r}\frac{\partial}{\partial x}\left(r\langle u_2u_2\rangle\right)+\frac{1}{r}\frac{\partial}{\partial r}\left(r\langle v_2u_2\rangle\right)+\frac{\partial\langle p_2\rangle}{\partial x}-\frac{1}{r}\frac{\partial}{\partial x}\left[r\left(2\frac{\partial\langle\mu u_2\rangle}{\partial x}-\frac{2}{3}\left(\frac{\partial\langle\mu u_2\rangle}{\partial x}+\frac{\partial\langle\mu v_2\rangle}{\partial r}+\frac{\mu v_2}{r}\right)\right)\right]$$

$$-\frac{1}{r}\frac{\partial}{\partial r}\left[r\left(\frac{\partial\langle\mu u_2\rangle}{\partial r}+\frac{\partial\langle\mu v_2\rangle}{\partial x}\right)\right]=\langle F_x\rangle, \tag{4.a}$$

$$\frac{1}{r}\frac{\partial}{\partial x}\left(r\langle\rho u_2v_2\rangle\right)+\frac{1}{r}\frac{\partial}{\partial r}\left(r\langle\rho v_2v_2\rangle\right)+\frac{\partial\langle p_2\rangle}{\partial r}-\frac{1}{r}\frac{\partial}{\partial x}\left[r\left(\frac{\partial\langle\mu v_2\rangle}{\partial x}+\frac{\partial\langle\mu u_2\rangle}{\partial r}\right)\right]$$

$$-\frac{1}{r}\frac{\partial}{\partial r}\left[r\left(2\frac{\partial\langle\mu v_2\rangle}{\partial r}-\frac{2}{3}\left(\frac{\partial\langle\mu u_2\rangle}{\partial x}+\frac{\partial\langle\mu v_2\rangle}{\partial r}+\frac{\langle\mu v_2\rangle}{r}\right)\right)\right]+2\frac{\langle\mu v_2\rangle}{r^2}$$

$$-\frac{2}{3}\frac{1}{r}\left(\frac{\partial\langle\mu u_2\rangle}{\partial x}+\frac{\partial\langle\mu v_2\rangle}{\partial r}+\frac{\langle\mu v_2\rangle}{r}\right)=\langle F_r\rangle, \tag{4.b}$$

$$\frac{\partial}{\partial x}\left(\langle\rho u_2\rangle\right)+\frac{\partial}{\partial r}\left(\langle\rho v_2\rangle\right)+\frac{\langle\rho v_2\rangle}{r}=\langle S_m\rangle. \tag{4.c}$$

Here angular brackets mean the time averaging. The terms of the second order F_x, F_r and S_m entering the right side are the result of the interaction of the first order terms:

$$F_x=-\rho_0\left\langle u_1\frac{\partial u_1}{\partial x}\right\rangle-\rho_0\left\langle v_1\frac{\partial u_1}{\partial r}\right\rangle-\left\langle\rho_1\frac{\partial u_1}{\partial t}\right\rangle, \tag{5.a}$$

$$F_r=0, \tag{5.b}$$

$$S_m=-\left\langle v_1\frac{\partial\rho_1}{\partial r}\right\rangle-\left\langle u_1\frac{\partial\rho_1}{\partial x}\right\rangle-\left\langle\frac{\rho_1}{\rho_0}\frac{\partial\rho_1}{\partial t}\right\rangle \tag{5.c}$$

One can determine F_x and S_m using (3):

$$F_x=-\frac{\rho_0r_1^2c_0\omega\tilde{r}}{4}\Big\{\sin\left(2k_0x(1+\beta')+2\alpha\right)\Big[\tilde{r}\left(1-2e^{-\eta}\cos\eta+2e^{-2\eta}\right)$$

$$+\frac{\kappa-1}{\sqrt{\sigma}}e^{-\left(1+\sqrt{\sigma}\right)\eta}\sin\left(1-\sqrt{\sigma}\right)\eta+\left(1+\frac{\kappa-1}{\sqrt{\sigma}}\right)\frac{r}{R}e^{-\eta}\sin\eta+1-e^{-\eta}\cos\eta$$

$$+(\kappa-1)e^{-\sqrt{\sigma}\eta}\cos\sqrt{\sigma}\eta-(\kappa-1)e^{-\left(1+\sqrt{\sigma}\right)\eta}\cos\left(1-\sqrt{\sigma}\right)\eta\Big]$$

$$+\sinh\left[2\left(\beta-k_0x\beta'\right)\right]\Big[2e^{-2\eta}+\frac{\kappa-1}{\sqrt{\sigma}}e^{-\left(1+\sqrt{\sigma}\right)\eta}\cos\left(1-\sqrt{\sigma}\right)\eta+e^{-\eta}\sin\eta$$

$$+(\kappa-1)e^{-\sqrt{\sigma}\eta}\sin\sqrt{\sigma}\eta+(\kappa-1)e^{-\left(1+\sqrt{\sigma}\right)\eta}\sin\left(1-\sqrt{\sigma}\right)\eta\Big]\Big\}, \tag{6.a}$$

$$
\begin{aligned}
S_m = \frac{r_1^2\omega\rho_0}{4}\Big\{&\cos\left[2k_0x(1+\beta')+2\alpha\right]\left[-\left(1+\frac{\kappa-1}{\sqrt{\sigma}}\right)(\kappa-1)\sqrt{\sigma}\,\frac{r}{R}e^{-\sqrt{\sigma}\eta}\cos\sqrt{\sigma}\eta\right.\\
&+(\kappa-1)\sqrt{\sigma}\,e^{-\left(1+\sqrt{\Pr}\right)\eta}\cos\left(1-\sqrt{\sigma}\right)\eta+(\kappa-1)^2e^{-2\sqrt{\sigma}\eta}+\tilde{r}\Big[e^{-\eta}\sin\eta+(\kappa-1)e^{-\sqrt{\sigma}\eta}\sin\sqrt{\sigma}\eta\\
&\left.+(\kappa-1)e^{-\left(1+\sqrt{\sigma}\right)\eta}\sin\left(1-\sqrt{\sigma}\right)\eta\Big]\right]+\cosh\left[2(\beta-k_0x\beta')\right]\left[-\left(1+\frac{\kappa-1}{\sqrt{\sigma}}\right)(\kappa-1)\sqrt{\sigma}\,\frac{r}{R}e^{-\sqrt{\sigma}\eta}\cos\sqrt{\sigma}\eta\right.\\
&+(\kappa-1)\sqrt{\sigma}\,e^{-\left(1+\sqrt{\sigma}\right)\eta}\cos\left(1-\sqrt{\sigma}\right)\eta+(\kappa-1)^2e^{-2\sqrt{\sigma}\eta}-\tilde{r}\Big[e^{-\eta}\sin\eta+(\kappa-1)e^{-\sqrt{\sigma}\eta}\sin\sqrt{\sigma}\eta\\
&\left.\left.+(\kappa-1)e^{-\left(1+\sqrt{\sigma}\right)\eta}\sin\left(1-\sqrt{\sigma}\right)\eta\Big]\right]\Big\}.
\end{aligned}
\tag{6.b}
$$

Equations (4) together with (6) describe secondary acoustic streaming in a cylindrical tube-resonator.

BOUNDARY CONDITIONS

All unknowns entering (4) and (6) can be determined from boundary conditions fixing the actual geometry. To test the proposed approach, we carry out simulation for a setup analogous to one that was used in experiments [8]. It consisted of a tube of 0.576 m length and of 26 mm inner diameter. The stack consisted of 3 layers 0.2 mm-thick Plexiglas plates with a plate-to-plate distance of 5 mm. It was located between half wavelength and $3/4$ wavelength i.e. between the loop and node of pressure standing wave in a tube without a stack. The stack length was 20 mm. The flow visualization was carried out at 1330-1340 Hz with pressure amplitude $p_1(x=0)$ about 242.4 Pa. A schematic view is given in figure1.

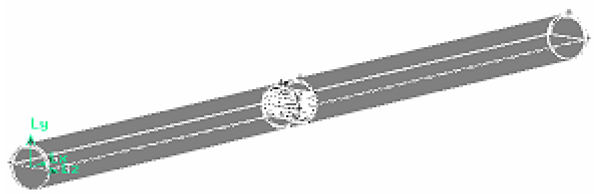


Figure1 – A schematic view of an experimental setup

Boundary conditions are needed at the source end and at the closed end, interface conditions in 8 sections in the cross-sections at x_1 and x_2 (x_1 and x_2 are the coordinates of stack ends), however, due to the axial symmetry, we have to formulate only 4.

The sound source is assumed to be a periodically oscillating piston (we shall call it "an open end"). For harmonic oscillation, the following relation is valid [6]:

$$u_1(x=0)=M_p e\left[i(\omega t-\pi/2)\right] \tag{7.a}$$

where $M_p = p_1(x=0)/\rho_0 c_0^2$ is the piston Mach number (for the present problem

$M_p = 0.0017$). (7.a) yields an expression for the magnitude r_{1op} and the phase ψ_{op} of the dimensionless oscillation amplitude in the area between the open end and a stack.

At a closed end, the velocity must be equal zero

$$\sin\{k_0 L[(1+\beta')+i\beta'']+\alpha_{cl}+i\beta_{cl}\}=0 \tag{7.b}$$

Separating real and imaginary parts, one obtains the relations to determine α_{cl} and β_{cl} through β' (subscript "cl" implies that the quantities correspond to the area between a stack and a closed end).

The other unknowns $\alpha_{st1,4}$, $\beta_{st1,4}$, r_{1cl}, $r_{1st1,4}$, $\psi_{st1,4}$, ψ_{cl}, $\alpha_{st2,3}$, $\beta_{st2,3}$, $r_{1st2,3}$, $\psi_{st2,3}$ α_{op}, β_{op} are determined from the continuity condition at the ends of the stack; the subscripts "st" correspond to the 4 areas inside the stack formed by the cross-sections x_1 and x_2 and stack plates, number 1,2,3,4 correspond to the sequential number of the area beginning from the bottom. We assume that the condition of continuous pressure and velocity must be valid at the tube-stack interface

$$p_{1op}\big|_{x=x_1} = p_{1st1,2,3,4}\big|_{x=x_1}, \quad u_{1op}\big|_{x=x_1} = u_{1st1,2,3,4}\big|_{x=x_1} \tag{7.c}$$

$$p_{1st1,2,3,4}\big|_{x=x_2} = p_{1cl}\big|_{x=x_2}, \quad u_{1st1,2,3,4}\big|_{x=x_2} = u_{1cl}\big|_{x=x_2} \tag{7.d}$$

Since expressions obtained from (7.c) and (7.d) are rather bulky, we do not give them here. The conditions (7.c) and (7.d) should be applied in every point of the cross-section. Instead of using the actual cross-sectional profile, the averaged profile over the cross-section of the corresponding area is used. This average normalized by the amplitude of duct velocity can be approximated for a cylindrical tube as follows [9]

$$B = 1-\sqrt{2}/H \tag{8.a}$$

Between the stack plates, instead of the radius R entering H the effective radius of a cross-section $R_{eff} = 2S/P$ is used, where S is the area and P is the perimeter of the corresponding cross-section. Effective radii for stack cross-sections are

$$R_{eff\ st1,4} = 0.003411; \ R_{eff\ st2,3} = 0.007560; \tag{8.b}$$

Solution of (7) subject to (8) yields

$$\begin{gathered}\beta'_{op} = 0.003225; \ \alpha_{op} = 4.809928; \ \beta_{op} = 0.051378; \\ B_{op} = 0.995616; \ \tilde{r}_{op} = 1.00323; \ r_{1op} = 0.001539 B_{op} \exp(-x); \\ \beta'_{st1,4} = 0.010890; \ \alpha_{st1,4} = 25.171641; \ \beta_{st1,4} = 0.110500;\end{gathered} \tag{9.a}$$

$$B_{st1,4} = 0.977322;\ \tilde{r}_{st1,4} = 1.010948;\ r_{1st1,4} = 0.001159 B_{st1,4} \exp(-10x); \tag{9.b}$$

$$\beta'_{st2,3} = 0.016770;\ \alpha_{st2,3} = 18.842306;\ \beta_{st2,3} = 0.156803;$$

$$B_{st2,3} = 0.996336;\ \tilde{r}_{st2,3} = 1.016908;\ r_{1st2,3} = 0.001145 B_{st2,3} \exp(-10x); \tag{9.c}$$

$$\beta'_{cl} = 0.003225;\ \alpha_{cl} = 6.382235;\ \beta_{cl} = 0.050177;$$

$$B_{cl} = 0.995616;\ \tilde{r}_{cl} = 1.00323;\ r_{1cl} = 0.000941 B_{cl} \exp(-10x). \tag{9.d}$$

Here the amplitude of a sound wave is assumed to decrease exponentially with the distance from a sound source.

RESULTS AND DISCUSSION

Simulation of the set (4) and (6) subject to (9) was carried out applying standard CFD methods making use of Fluent software. At the start of the simulations all variables throughout the simulation domain were set to zero $u_2 = 0,\ \ v_2 = 0,\ \ p_2 = 0,\ \ T_2 = 0$. In the case of transient flows, it is necessary sometimes to start from asymmetrical initial conditions to obtain certain stable solutions. In the example at hand, we introduced an initial velocity in the small bottom area in the stack $u_2 = -3\ m/s$. Under these conditions, a stable solution is obtained, even if high numerical accuracy is requested.

The damping and distorting effect of a stack on secondary streaming can easily be identified by comparing secondary streaming calculated for a closed cylindrical tube without a stack and the numerical results for the full device, including the stack. A fragment of streamlines in an axial view is presented in fig. 2.

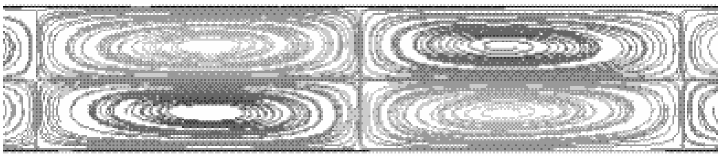
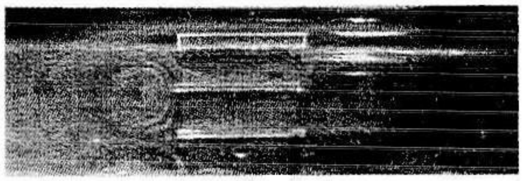


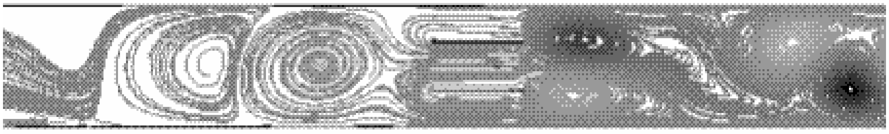
Figure2 – Secondary streaming in a closed tube

Their form coincides with the results of known studies [5, 7]. Streamlines in the model with the stack are given in fig. 3.b. One can see that streaming is strongly damped within the stack area and that a vortex occurs behind the stack. This agrees with pictures obtained from visualization of streaming in [8] (fig.3.a).

It is seen from the figures that this vortex motion results in hindering the further propagation of streaming and diverting the fluid back onto the stack. One can assume that this circulation motion through the stack has a non-negligible effect on heat pumping.



3.a



3.b

Figure 3 – Particle path lines obtained: 3.a – in experiment [8]; 3.b – in simulation

SUMMARY

Simulation of secondary streaming was carried out for an acoustic-resonance tube refrigerator and the effect of a stack on streaming was examined. A specific numerical model has developed for this purpose. The model combines analytical solution of the acoustic part of the field with a standard CFD computation for the secondary, streaming part of the fluid. Obtained particle path lines show that a stack decelerates the flow and distorts the symmetry of streaming. Results of the simulation agree with known experimental data. The effect of this streaming on the efficiency of heat pumping could not be quantified by the time of writing of this manuscript.

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Full paper (PDF format).

Abstract

A 3D numerical model of secondary streaming in an acoustic-resonance tube refrigerator filled with viscous, heat conducting air, is presented. It is known that secondary streaming is described by the second order time-average velocity; therefore second order steady-state Navier-Stokes equations are solved to obtain time-averaged thermodynamic quantities of the second order approximation. The first order quantities that enter the equations of the second order as source terms are derived from analytical solutions of the (acoustic) first order Navier-Stokes equations for a viscous heat conducting medium. The second order equations are solved numerically by virtue of a standard CFD method using accurate boundary conditions at the tube ends and interface conditions in the cross-sections separating stack and tube-resonator. Particle path lines are obtained and their general form is compared with known experimental data on the visualization of secondary streaming. The model attempts to explain the effect of a stack on secondary streaming and heat pumping in an acoustic-resonance tube refrigerator. In particular, it shows that the propagation of streaming is hindered and its symmetrical form is distorted due to the stack. Vortex motion occurs behind the stack resulting in circulation motion through the stack.